# Power and preferences: an experimental approach

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- in boards of directors (do we invest or not?)
- etc...

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- Player $i \in S$ is *pivotal* in the coalition $S$ iff $S$ is winning, while $S \setminus \{i\}$ is not

# Classical power indices

- Banzhaf (1965): $\beta_i = \frac{\sum_{S \subseteq N}(v(S) - v(S \setminus \{i\}))}{\sum_{j=1}^{N} \sum_{S \subseteq N}(v(S) - v(S \setminus \{j\}))} = \frac{b_i}{\sum_j b_j}$

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- Shapley-Shubik (1954):
  $\phi_i = \sum_{S \subseteq N} \frac{(|S|-1)!(N-|S|)!}{N!} (v(S) - v(S \setminus \{i\}))$.
  This is the share of permutations of all coalitions $S$ in which player $i$ is pivotal in the total number of permutations in which any player is pivotal, i.e. the Shapley value for the cooperative voting game.

# Example

Suppose $N = \{1, 2, 3\}, w_1 = 50, w_2 = 45, w_3 = 5, q = 51$. Then

$W = \{\{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$

$b_1 = 3, b_2 = 1, b_3 = 1$.

In this example,

Banzhaf $\beta_1 = 3/5; \beta_2 = \beta_3 = 1/5$.

Shapley-Shubik $\phi_1 = 2/3; \phi_2 = \phi_3 = 1/6$.

# Power indices with preferences

(Aleskerov (2006)). Assume we know the preference profile of each player $i$ about coalescing with any other player: $P_i = (p_{i1}, ..., p_{in})$.

Let $p_{ij}$ be (ordinal or cardinal) *measures of preference*, or *explicit modifiers* of player $i$ towards coalescing with player $j$.

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Let $\chi_i = \sum_S f_i(S)\,(v(S) - v(S \setminus \{i\}))$ be the sum of intensities of connection of player $i$ over all the winning coalitions in which she is pivotal. Similarly to the Banzhaf index

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$$\alpha_i = \frac{\sum_{S \subseteq N} f_i(S)(v(S) - v(S \setminus \{i\}))}{\sum_{j=1}^{N} \sum_{S \subseteq N} f_j(S)(v(S) - v(S \setminus \{i\}))} = \frac{\chi_i}{\sum_{j=1}^{N} \chi_j}$$

# Power indices with preferences

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- ... and many others.

# Power indices with preferences: Example

$N = \{1,2,3\}, w_1 = 50, w_2 = 45, w_3 = 5, q = 51.$

$W = \{(1,2),(1,3),(1,2,3)\}$

Assume preferences are cardinal and given by:

$$||p_{ij}|| = \left\{ \begin{array}{|c|ccc|} \hline & 1 & 2 & 3 \\ \hline 1 & & \frac{1}{2} & 2 \\ 2 & 1 & & 1 \\ 3 & 2 & 1 & \\ \hline \end{array} \right.$$

Product intensities of connections across coalitions are

| | $(1,2)$ | $(1,3)$ | $(1,2,3)$ |
|---|---|---|---|
| *player1* | $f_1^{\times}(1,2) = 1/2$ | $f_1^{\times}(1,3) = 2$ | $f_1^{\times}(1,2,3) = (\frac{1}{2} \cdot 2)/2 = 1/2$ |
| *player2* | $f_2^{\times}(1,2) = 1$ | | $f_2^{\times}(1,2,3) = (1 \cdot 1)/2 = 1/2$ |
| *player3* | | $f_3^{\times}(1,3) = 2$ | $f_3^{\times}(1,2,3) = (2 \cdot 1)/2 = 1$ |

# Power indices with preferences: example continued

Sums of intensities of connections over the winning coalitions:

Player 1 is pivotal in 3 coalitions, so

$\chi_1 = f_1^{\times}(1,2) + f_1^{\times}(1,3) + f_1^{\times}(1,2,3) = 1/2 + 2 + 1/2 = 3$

Player 2 is pivotal in 2 coalitions, so

$\chi_2 = f_2^{\times}(1,2) + f_2^{\times}(1,2,3) = 1 + 1/2 = 3/2$

Player 3 is pivotal in 2 coalitions, so

$\chi_3 = f_3^{\times}(1,3) + f_3^{\times}(1,2,3) = 2 + 1 = 3$

The generalized power indices are:

$$\alpha_1 = \frac{\chi_1}{\sum_{i=1}^{3} \chi_i} = 2/5,$$
$$\alpha_2 = \frac{\chi_2}{\sum_{i=1}^{3} \chi_i} = 1/5,$$
$$\alpha_3 = \frac{\chi_3}{\sum_{i=1}^{3} \chi_i} = 2/5.$$

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- ... and many others.

# Previous works

Brams and Affuso, TD, 1976 notice that adding an extra player affects power indices of the remaining ones ('paradox of the new members').

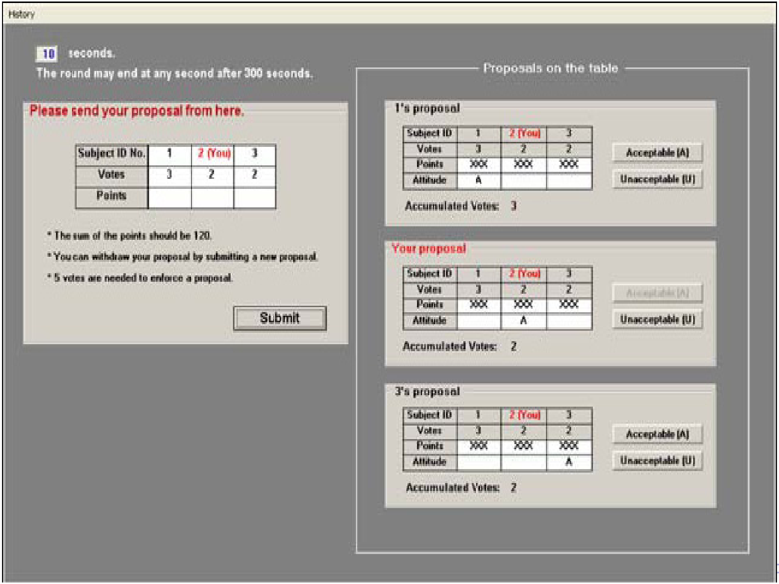
Kahan and Rapoport, 1984 summarise theoretical and empirical studies of voting in the context of cooperative games.

Selten Kuon, IJGT, 1993 study dynamic bargaining in three-person games

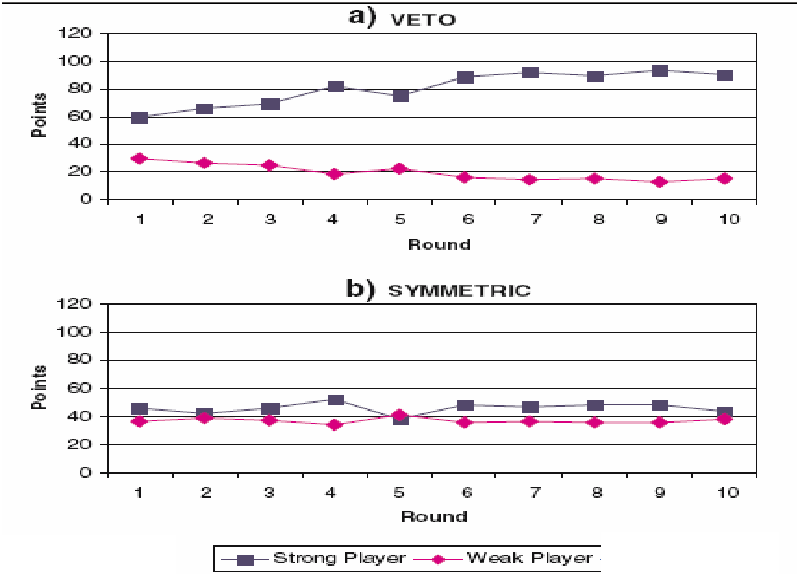
Kagel e.a. 2009 explore the role of veto power

Montero, Sefton, Zhang, Soc.Ch.Welf., 2008 test the paradox of new members experimentally.

# Game Standard (MSZ)



# Outcomes (MSZ)



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- In case the players do not come to an agreement within 300 seconds, they receive 0 pts
- Each game lasts 10 or 20 rounds (enlarged games), all players are mixed in roles and across groups in each round
- 2 games are played in each experimental session in randomized block order. At the end of each session, total gains are paid in cash (1 point = 0.01 EUR in Russian Rubles).

# Participants

- 136 BSc and MSc students of various departments of HSE.
- Recruitment through posters and announcement on the web, volunteers are requested to register online. Subjects are invited by email to a particular game.
- Attendance: required additional recruitment on-site.
- Gender composition: about 50:50, average age — 19.1 years
- Gains of participants in 10-round games: average 7.62 EUR, minimum — 3.81 EUR, maximum — 13.68 EUR;
- Gains in 20-round games: average 10.65 EUR, minimum 5.38 EUR, maximum 16.81 EUR per 1- to 1.5-hour session.

# Summary of experimental sessions

| 1st game | 2nd game |
|---|---|
| S | 1 |
| 1 | V |
| 2 | S |
| SC | 1C |
| 1C | 2 |
| V | SC |
| E | 3 |
| 3 | E |
| F | 4 |
| 4 | F |

Games V–2,E–3,F–4 (S–1): the $2 \times 2(\times 2)$ design, controlling for

1. sequence of the games
2. explicit modifiers
3. position of players on the screen (C)

# Game Standard (S)

| Player number | 1 | 2 (You) | 3 |
|---|---|---|---|
| Votes | 3 | 2 | 2 |
| Proposed shares | 60 | 60 | |

Submit your proposal

- Instruction
- Shares should sum up to 120
- You can replace your proposal by a newly submitted one
- 4 votes are required to pass a proposal
- You are marked in red where applicable
- Please note that your login name shown at the bottom of this page is NOT your in-game player number! Also beware that your in-game player number may change between rounds

121 seconds left

**Player #2's proposal (Total votes accumulated: 2)**

| Player number | 1 | 2 (You) | 3 | |
|---|---|---|---|---|
| Votes | 3 | 2 | 2 | You have voted for this proposal |
| Proposed shares | 45 | 75 | 0 | |
| Acceptance | | Y | | |

**Player #3's proposal (Total votes accumulated: 2)**

| Player number | 1 | 2 (You) | 3 | |
|---|---|---|---|---|
| Votes | 3 | 2 | 2 | Vote for this proposal |
| Proposed shares | 20 | 30 | 70 | |
| Acceptance | | | Y | |

# Game Standard with modifiers (1)

| Player number | 1 | 2 (You) | 3 |
|---|---|---|---|
| Votes | 3 | 2 | 2 |
| Proposed shares | | | |

Submit your proposal

- Instruction
- Shares should sum up to 120
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- 4 votes are required to pass a proposal
- You are marked in red where applicable
- Please note that your login name shown at the bottom of this page is NOT your in-game player number! Also beware that your in-game player number may change between rounds
- If you vote for a proposal and it wins, for each of the players voting together with you, your share in the proposal will be multiplied by the corresponding "modifier" value(s) from the table below to get your final payoff in this round.

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | — | 1 | 1 |
| 2 | 1 | — | 1.01 |
| 3 | 1 | 1 | — |

55 seconds left

**Player #2's proposal (Total votes accumulated: 2)**

| Player number | 1 | 2 (You) | 3 | |
|---|---|---|---|---|
| Votes | 3 | 2 | 2 | You have voted for this proposal |
| Proposed shares | 20 | 60 | 40 | |
| Acceptance | | Y | | |

**Player #3's proposal (Total votes accumulated: 2)**

| Player number | 1 | 2 (You) | 3 | |
|---|---|---|---|---|
| Votes | 3 | 2 | 2 | Vote for this proposal |
| Proposed shares | 35 | 25 | 60 | |
| Acceptance | | | Y | |

# Games S–1 (Standard)

**Game S**: In this game **4** votes are required to reach an agreement

| player# | 1 | 2 | 3 |
|---|---|---|---|
| votes | 3 | 2 | 2 |

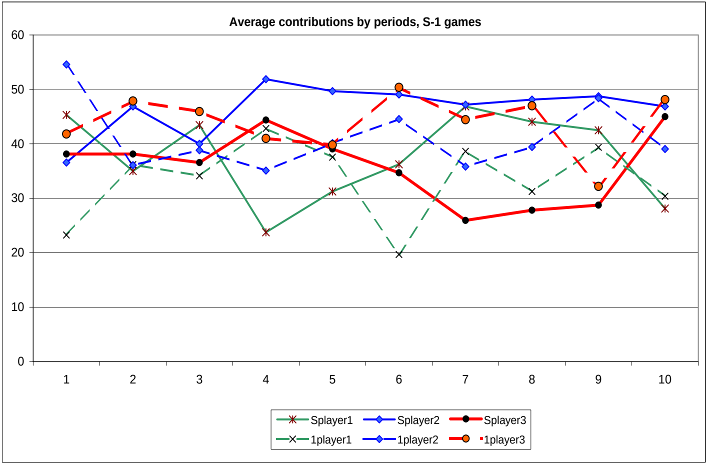
Winning coalitions: $W = \{\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$. Banzhaf index: $\beta_1 = \beta_2 = \beta_3 = 1/3$, predicting that all players get around **40 pts** each. **Game 1** uses the following *explicit modifiers* which multiply the payoff of the row player if she coalesces with the column player:

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | - | 1 | 1 |
| 2 | 1 | - | 1.01 |
| 3 | 1 | 1 | - |

$\alpha$ indices based on the $f^{\times}$ intensity function:

$\alpha_1 = \alpha_3 = 0.3327, \alpha_2 = 0.3344$

# Results: S-1 games



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- Player 3 on average receives systematically more in the 1–treatment (42.08) than in S–treatment (32.25), which difference is significant. Hence **explicit modifiers** do work for player 3: '*being loved is better than love*'.
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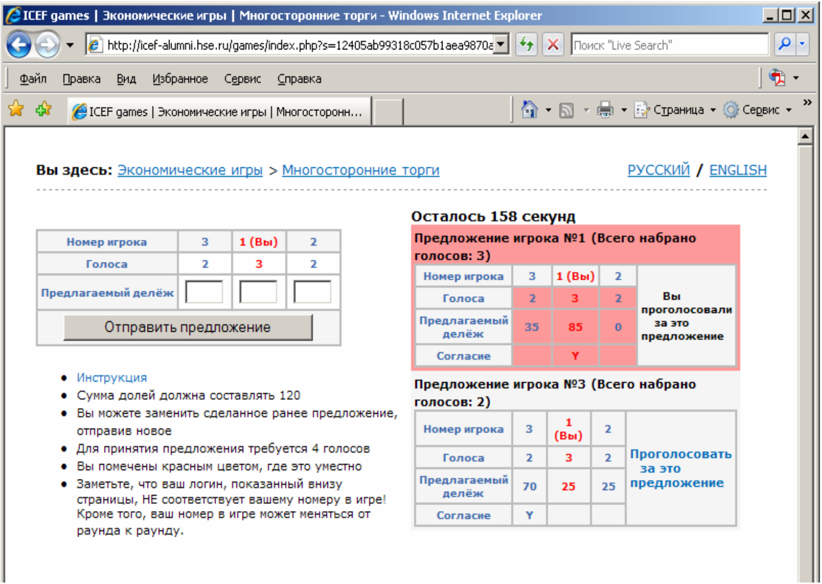
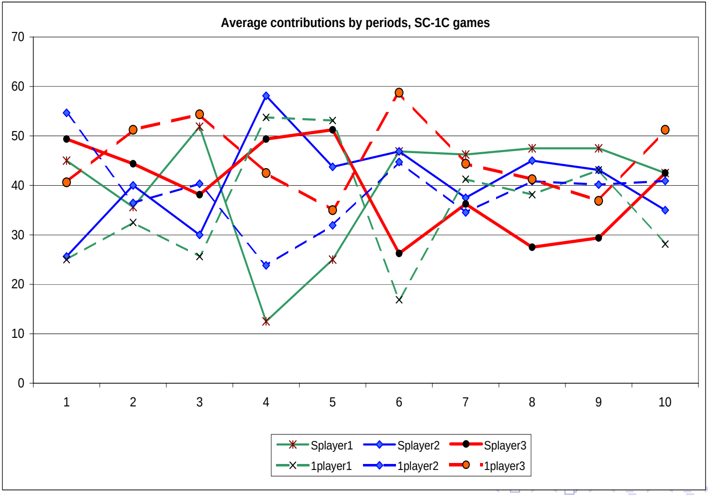
- There are no treatment effects for players 1 and 2, but ...
- Player 2 receives systematically more than player 3 in both treatments combined (49.89 vs. 37.14), which difference is significant.
  - Same effect as in MSZ, who attribute it to 'framing effect'
  - We attribute it to the position of player 2 in the middle of the table on the screen: player 2 has two neighbours (1 and 3), whereas the other two players — just one (player 2). We refer to this effect as to the **implicit modifier** to player 2's payoff.

# Game Standard Centered (SC)



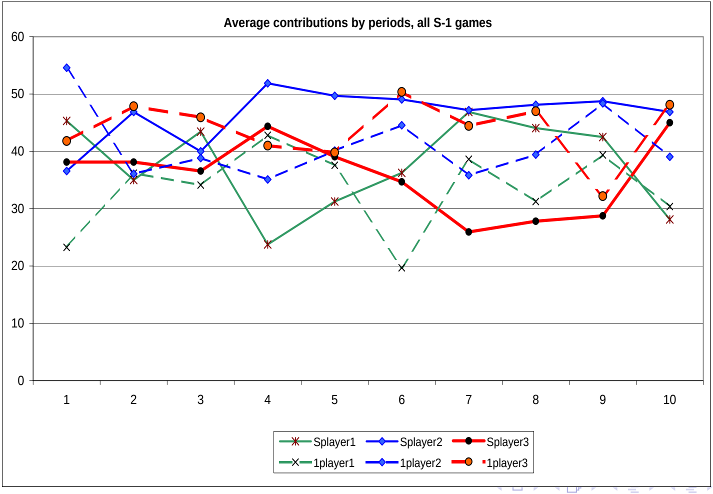
# Results: SC–games



# Way out: symmetric positioning

- In SC-1C games, each player is shown in the middle of the table in a systematic (clockwise) rotation.
- The difference between players 2 and 3 is mitigated from (49.89 vs. 37.14) to (43.03 vs. 39.41), and becomes insignificant
  - The effect of *implicit modifier* is most likely to completely disappear in a fully symmetric treatment, but we suppose this is not very interesting, being a feature of a particular experiment.
- *Explicit modifiers*' effect persists for player 3.
- Average number of offers in games S (1) — 2.13 (resp., 2.42).
- Average decision time in games S (1) — 30 (resp., 37) seconds.

# Results: all S–1 games



# Summary of the S-1 games

| All ($N = 320$) | mean | s.d. | min | max |
|---|---|---|---|---|
| player 1 | 35.36 | 29.04 | 0 | 80 |
| player 2 | 44.53 | 24.42 | 0 | 100 |
| player 3 | 40.1 | 27.56 | 0 | 111 |
| Game S | | | | |
| player 1 | 37.40 | 29.44 | 0 | 80 |
| player 2 | 46.25 | 23.89 | 0 | 100 |
| player 3 | **36.34** | 28.05 | 0 | 110 |
| Game 1 | | | | |
| player 1 | 33.32 | 28.57 | 0 | 80 |
| player 2 | 42.81 | 24.91 | 0 | 99 |
| player 3 | **43.85** | 26.62 | 0 | 111 |

- No significant difference in payoffs for players 1 and 2.
- Significant difference for player 3 at 1-2% confidence level.
- Centered treatment suppresses implicit modifiers.

# Games V–2 (Veto)

**Game V**: Now **5** votes are required to reach an agreement

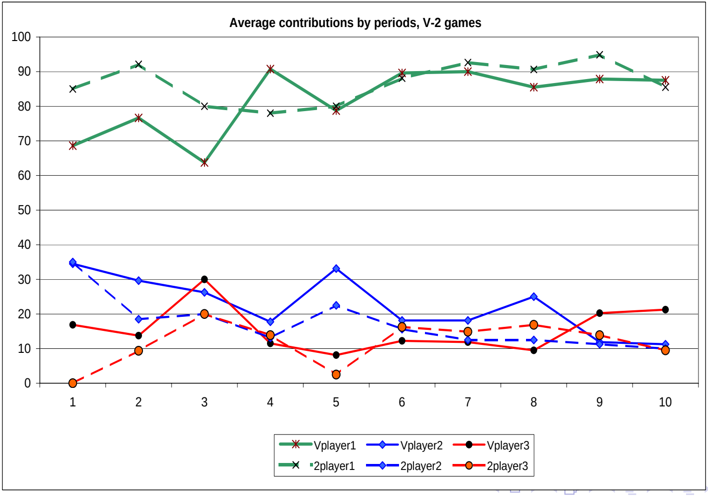
| player# | 1 | 2 | 3 |
|---|---|---|---|
| votes | 3 | 2 | 2 |

Winning coalitions $W = \{\{1,2\},\{1,3\},\{1,2,3\}\}$. Banzhaf: $\beta_1 = 3/5, \beta_2 = \beta_3 = 1/5$ predicts that player 1 gets 72 pts and players 2 and 3 — 24 pts each. **Game 2** uses the following *explicit modifiers*:

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | - | 1 | 1 |
| 2 | 0.99 | - | 1 |
| 3 | 0.99 | 1 | - |

$\alpha$ indices based on the $f^{\times}$ intensity function:
$\alpha_1 = 0.5575, \alpha_2 = \alpha_3 = 0.2212$

# Results: V–2 games



# Results: V–2 games

- Player 1 (the veto player) gets even more than the Banzhaf index predicts.
- No significant difference across treatments.
- Effects of greater negative modifiers might be larger.
- Average number of offers in games V (2) — 5.94 (resp., 5.63).
- Average decision time in games V (2) — 147 (resp., 141) seconds. Timing of decisions requires further attention.

# Games E–3 (Enlarged)

**Game E**: Again, **5** votes are required to reach an agreement

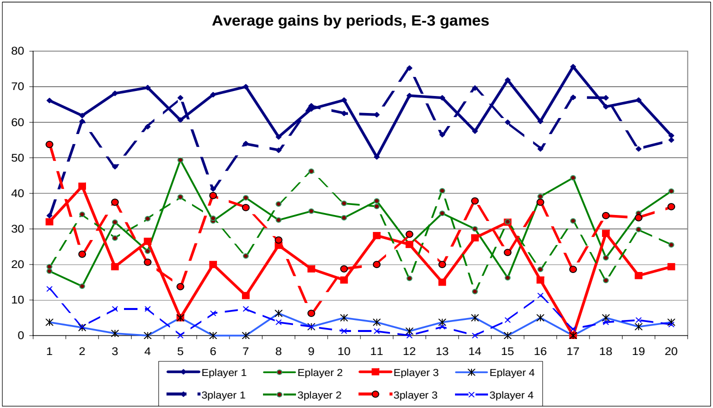
| player# | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| votes | 3 | 2 | 2 | 1 |

Winning coalitions $W = \{\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\}$. Here, $\beta_1 = 5/12, \beta_2 = \beta_3 = 3/12, \beta_4 = 1/12$, predicted payoffs are (50, 30, 30,10). **Game 3** employs the following modifiers:

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | - | 1 | 1 | 1 |
| 2 | 0.99 | - | 1 | 1 |
| 3 | 1 | 1 | - | 1 |
| 4 | 1 | 1 | 1 | - |

$\alpha$ indices based on the $f^{\times}$ intensity function:
$\alpha_1 = 0.5007, \alpha_2 = 0.2131, \alpha_3 = 0.2212, \alpha_4 = 0.0715$

# The E–3 games



Average gains by periods, E-3 games
80
70
60
50
40
30
20
10
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Eplayer 1
Eplayer 2
Eplayer 3
Eplayer 4
3player 1
3player 2
3player 3
3player 4

# Summary of the E-3 games

| All ($N = 160$) | mean | s.d. | min | max |
|---|---|---|---|---|
| player 1 | 61.15 | 25.76 | 0 | 100 |
| player 2 | 30.63 | 23.82 | 0 | 70 |
| player 3 | 24.73 | 24.54 | 0 | 70 |
| player 4 | 3.49 | 9.00 | 0 | 70 |
| Game E | | | | |
| player 1 | **64.34** | 22.36 | 0 | 95 |
| player 2 | 31.65 | 23.17 | 0 | 70 |
| player 3 | **21.23** | 23.72 | 0 | 70 |
| player 4 | 2.76 | 7.40 | 0 | 40 |
| Game 3 | | | | |
| player 1 | **57.95** | 28.47 | 0 | 100 |
| player 2 | 29.59 | 24.48 | 0 | 70 |
| player 3 | **28.23** | 24.90 | 0 | 65 |
| player 4 | 4.21 | 10.33 | 0 | 70 |

# Results: E-3 games

- In **E-game** player 1 gets systematically more than the Banzhaf index prediction at the expense of player 4, while gains of players 2 and 3 are in line with the index, and are *greater than in the V–2 treatment*.

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    - Thus, a small negative modifier towards player 1 indirectly benefits player 3 (gain per treatment increases by 25%).

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- Player 3 gains significantly more on average in the 3-treatment.
  - Thus, a small negative modifier towards player 1 indirectly benefits player 3 (gain per treatment increases by 25%).
- Frequency of coalitions $\{2, 3, 4\}$ is two times higher in the 3–game than in the E–game.
  - Means that players 2, realizing they do not like player 1, tend to switch to a larger coalition, even though it is clearly more difficult and may involve lowering one's share of the pie (has to be divided among 3 players instead of 2 ).

# Games F–4

**Game F**: 6 votes required to reach an agreement

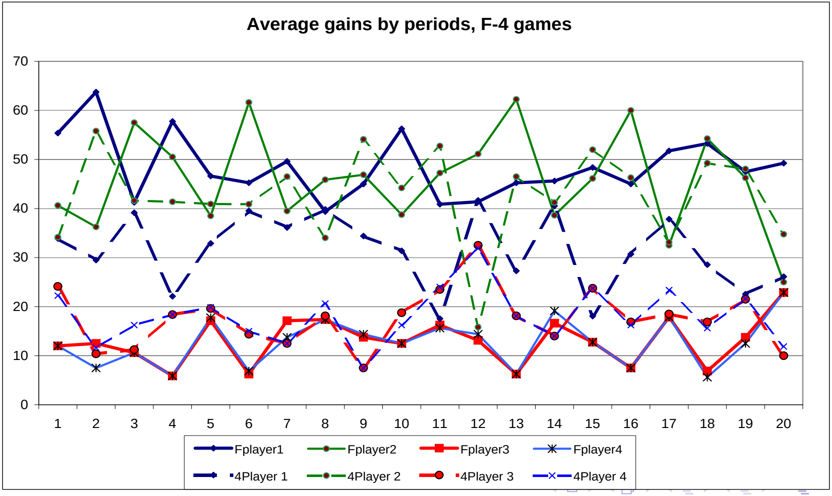
| player# | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| votes | 3 | 3 | 2 | 2 |

Winning coalitions
$W = \{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\}$.
Banzhaf index is $\beta_1 = \beta_2 = 1/3, \beta_3 = \beta_4 = 1/6$, 1 and 2 get 40, 3 and 4 get 20 each. **Game 4** employs the following modifiers:

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | - | 0.8 | 1 | 1.01 |
| 2 | 0.8 | - | 1 | 1.1 |
| 3 | 1 | 1 | - | 1 |
| 4 | 1 | 1 | 1 | - |

$\alpha$ indices based on $f^{\times}$ intensity function:
$\alpha_1 = 0.3348, \alpha_2 = 0.3476, \alpha_3 = \alpha_4 = 0.1587$.

# The F–4 games


Average gains by periods, F-4 games
70
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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Fplayer1
Fplayer2
Fplayer3
Fplayer4
4Player 1
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# F-game vs. 4-game

- 'Large' negative modifier of player 2 for player 1 significantly lowers her earnings (48.43 vs 31.48).
- On the contrast, player 2's payoff does not change much.
- Complex interaction of modifiers: high 'dislike' modifiers of 0.8 tend to hurt player 1 more than player 2 because player 2 more strongly prefers larger coalitions.
- Another explanation we investigated – that the *psychological features* of the subjects' characters.

# Summary of the F–4 games

| All ($N = 160$) | mean | s.d. | min | max |
|---|---|---|---|---|
| player 1 | 39.95 | 10.97 | 17.55 | 63.75 |
| player 2 | 44.32 | 9.68 | 15.81 | 62.25 |
| player 3 | 15.24 | 5.75 | 5.88 | 32.50 |
| player 4 | 15.35 | 5.87 | 5.63 | 31.88 |
| Game F | | | | |
| player 1 | **48.43** | 6.34 | 39.38 | 63.75 |
| player 2 | 45.97 | 10.02 | 25.00 | 62.25 |
| player 3 | **12.95** | 4.68 | 5.88 | 22.88 |
| player 4 | **12.66** | 4.95 | 5.63 | 22.88 |
| Game 4 | | | | |
| player 1 | **31.48** | 7.46 | 17.55 | 41.66 |
| player 2 | 42.67 | 9.30 | 15.81 | 55.80 |
| player 3 | **17.53** | 5.90 | 7.50 | 32.50 |
| player 4 | **18.04** | 5.58 | 7.50 | 31.88 |

# Coalitional outcomes across treatments

| coalitions ╲ games | S–1 games | | | V–2 games | |
|---|---|---|---|---|---|
| S-1 coalitions | S | 1 | V-2 coalitions | V | 2 |
| **1&2** | 54 | 33 | **1&2** | 41 | 40 |
| **2&3** | 29 | 33 | **2&3** | 27 | 26 |
| **2&3** | 56 | 59 | 1&2&3 | 12 | 10 |
| 1&2&3 | 21 | 35 | 1 alone | 0 | 1 |
| other | 0 | 0 | none | 0 | 3 |
| total | 160 | 160 | total | 80 | 80 |

| coalitions ╲ games | E–3 games | | | F–4 games | |
|---|---|---|---|---|---|
| E-3 coalitions | E | 3 | F-4 coalitions | F | 4 |
| **1&2** | 73 | 74 | **1&2** | 82 | 64 |
| **2&3** | 57 | 51 | **1&3&4** | 38 | 31 |
| **2&3&4** | 13 | 26 | **2&3&4** | 33 | 56 |
| 1&2&3 | 5 | 1 | 1&2&3 | 1 | 1 |
| 1&2&4 | 1 | 3 | 1&2&4 | 1 | 0 |
| 1&3&4 | 1 | 1 | 1&3 | 0 | 1 |
| 1&2&3&4 | 9 | 3 | 1&4 | 0 | 0 |
| none | 1 | 0 | 1&2&3&4 | 4 | 6 |
| total | 160 | 160 | total | 160 | 160 |

# Empirical Banzhaf indices: S–1 games

| index | Banzhaf | S | SC | S all | 1 | 1C | 1 all |
|---|---|---|---|---|---|---|---|
| 1 | 40 | 32.81 | 38.40 | 35.82 | 28.96 | 34.03 | 31.68 |
| 2 | 40 | 54.37 | 41.60 | 47.48 | 48.62 | 40.30 | 44.16 |
| 3 | 40 | 32.81 | 40.00 | 36.69 | 42.41 | 45.67 | 44.16 |
| factual | | | | | | | |
| 1 | | 35.25 | 40.06 | 37.66 | 30.90 | 35.75 | 33.33 |
| 2 | | 52.50 | 40.50 | 46.50 | 47.28 | 38.83 | 43.06 |
| 3 | | 32.25 | 39.44 | 35.84 | 42.93 | 45.62 | 43.86 |

# Empirical Banzhaf indices: V, E and F–games

| | V–2 games | | | E–3 games | | | F–4 games | | |
|---|---|---|---|---|---|---|---|---|---|
| index | B | V | 2 | B | E | 3 | B | F | 4 |
| 1 | 72 | 64.9 | 64.2 | 50 | 53.4 | 46.3 | 40 | 38.4 | 29.5 |
| 2 | 24 | 33.2 | 33.8 | 30 | 33.9 | 36.7 | 40 | 36.8 | 37.1 |
| 3 | 24 | 21.9 | 22.0 | 30 | 27.7 | 27.7 | 20 | 22.4 | 26.7 |
| 4 | | | | 10 | 5.0 | 9.3 | 20 | 22.4 | 26.7 |
| factual | | | | | | | | | |
| 1 | | 84.6 | 86.7 | | 64.3 | 57.9 | | 48.4 | 31.7 |
| 2 | | 21.2 | 17.0 | | 31.7 | 29.4 | | 46.0 | 43.1 |
| 3 | | 14.2 | 11.6 | | 21.3 | 28.2 | | 13.0 | 18.0 |
| 4 | | | | | 2.8 | 4.2 | | 12.7 | 18.4 |

The explanation seems to be in unequal prior probability of all coalitions, calling for the use of extended power indices over the standard ones.

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- Implicit modifiers in the S–game can be suppressed by centering the players and other means.
- Explicit modifiers (probably) do not work in the V–2 games.
- The intensity of connections of other players to the given player *i* (probably) matters more for her payoff: 'being loved is better than love'.

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- Modifiers of opposite nature interact in a complex manner.
- Predictive power of the classical power indices is ambiguous: the best explanatory variables are player numbers and winning coalitions.

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# Q & A

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